Location Estimation for an Autonomously Guided Vehicle using an Augmented Kalman Filter to Autocalibrate the Odometry

Thomas Dall Larsen, Martin Bak, Nils A. Andersen & Ole Ravn
Department of Automation, Technical University of Denmark
Building 326, DK-2800 Lyngby, Denmark
e-mail: or@iau.dtu.dk

Abstract

A Kalman filter using encoder readings as inputs and vision measurements as observations is designed as a location estimator for an autonomously guided vehicle (AGV). To reduce the effect of modelling errors an augmented filter that estimates the true system parameters is designed. The traditional way of reducing these errors is by fictitious noise injection in the filter model. The main problem with that approach however is that the filter does not learn about its bad model, it just puts more confidence in incoming measurements and less in the model. As a result the estimates will drift and the covariance grow rapidly between measurements causing these to be fused at a very high gain. This not only leads to a very "bumpy" behavior of the estimates and a high sensitivity to measurement noise but will also lead to large estimation errors in the absence of measurements. The taken approach offers a better suppression of vision measurement noise and a better performance in the absence of vision measurements.

Keywords: Kalman, fusion, odometry, modelling, autocalibration

1 Introduction

In AGV navigation one of the key problems is how to estimate the location of the AGV. By far the most common way of obtaining this estimate is by using a Kalman filter. The Kalman filter uses a system model along with measurements from internal and external sensors to maintain an estimate of the AGVs location and of a corresponding covariance matrix describing the uncertainty of the location estimate. Knowing the covariance matrices of the estimate and the incoming measurements, the filter fuses measurements and estimate, minimizing the covariance of the resulting estimate.

In order for the estimate to remain optimal it is required that the model of the AGV is perfect and noise influencing the system and measurements are white, zero mean, additive and Gaussian, see [1]. As these conditions rarely are fulfilled, most Kalman filters run in a suboptimal and possibly unstable manner. The most common way in practice to prevent an erroneous filter model to bias or diverge the estimates is to force the filter to put less confidence in the model and more in the measurements. This is done by increasing the filters process noise covariance matrix, Q, which is equivalent to adding fictitious process noise to simulate the uncertainties.

This *tuning* of Q is almost always necessary as it is impossible to model a real AGV perfectly no matter how advanced the model is chosen. In fact if the AGV is equipped with an odometric system a better possibility will often be simply to use the odometry outputs as the system model. For the common case when the odometry system consists of two encoders a model has been derived in [2] that has been adopted by a large number of authors for instance [3] or [4].

This model is quite simple and has only three physical parameters that needs to be known precisely. Knowing these are essential for good filter performance and various methods for determining them have been proposed. In [5] a method is presented where a sequence of AGV runs is used to determine the parameters. The method requires that the position of the AGV is calibrated manually with great accuracy.

A more automized approach is presented in [6] where an indirect continuous Kalman filter is designed to estimate the errors on these three parameters (along with some additional ones due to a gyroscope). In this paper an approach is taken where the model is augmented with a correction factor for these three parameters. When the filter relies solely on encoder measurements the filter behaves as a non-augmented filter (except that it updates an augmented covariance matrix). When a measurement is fused this will not only correct the location estimate but will also improve the filter model.

2 The Kalman filter

The AGV is equipped with an encoder on each motor shaft. An angular increment of α on the encoder corresponds to a distance, d, on the wheel periphery, where $d = k\alpha$. If the movement of the AGV is assumed linear the distances d_r and d_l traveled by the right and the left wheel respectively, can be transformed to a linear and angular displacement of the AGV:

$$\Delta d_k = \frac{d_{r,k} + d_{l,k}}{2} \quad (1)$$

$$\Delta \theta_k = \frac{d_{r,k} - d_{l,k}}{b} \quad (2)$$

where b is the distance between the wheels.

The AGVs coordinates in a global coordinate frame can then be updated by (see [2]):

$$X_{k+1} = X_k + \Delta d_k \cos(\theta_k + \frac{\Delta\theta_k}{2}) \quad (3)$$

$$Y_{k+1} = Y_k + \Delta d_k \sin(\theta_k + \frac{\Delta\theta_k}{2}) \quad (4)$$

$$\theta_{k+1} = \theta_k + \Delta\theta_k \quad (5)$$

These three coordinates constitutes the AGV state vector x, and are observed by vision measurements, z. The vision measurements can be described by a nonlinear function, c, of the AGV coordinates and an independent Gaussian noise process, v. Denoting the nonlinear function (3)-(5) a and collecting Δd_k and $\Delta\theta_k$ in an input vector u_k the AGV can be described by equation (6)-(7).

$$x_{k+1} = a(x_k, u_k, w_k, k) \quad (6)$$

$$z_k = c(x_k, v_k, k), \quad (7)$$

where: $w_k \sim N(0, Q_k)$ and $v_k \sim N(0, r_k)$ and $E\left[w_i v_j^T\right] = 0$.

An extended Kalman filter can be designed using this system model (as in [4]). The filter equations are summarized below in (8) to (12):

$$\hat{x}_{k+1} = a(\hat{x}_k, u_k, 0, k) \quad (8)$$

$$P_{k+1} = A_k P_k(+) A_k^T + G_k Q_k G_k^T \quad (9)$$

$$K_k = P_k C_k^T \left[C_k P_k C_k^T + R_k\right]^{-1} \quad (10)$$

$$\hat{x}_k(+) = \hat{x}_k + K_k \left[z_k - C_k \hat{x}_k\right] \quad (11)$$

$$P_k(+) = \left[I - K_k C_k\right] P_k \quad (12)$$

where:

$$A_k = \left.\frac{\partial a}{\partial \hat{x}_k}\right|_{u_k, w_k = 0}, \; G_k = \left.\frac{\partial a}{\partial w_k} Q_k \frac{\partial a}{\partial w_k}^T\right|_{u_k, \hat{x}_k}$$

$$C_k = \left.\frac{\partial c}{\partial \hat{x}_k}\right|_{v_k = 0}, \quad R_k = \left.\frac{\partial c}{\partial v_k} r_k \frac{\partial c}{\partial v_k}^T\right|_{\hat{x}_k}$$

The process noise vector is modelled as two independent Gaussian white noise processes added to (1) and (2). As shown in [2] this is a fairly good approximation.

3 Augmented Kalman Filter

The main problem with the Kalman filter described in section 2 is its sensitivity to modelling errors. If the distance between the wheels or the wheel radiuses and gearings are not precisely known, the time update (8) will lead the estimates to drift. In the absence of vision measurements this could cause the estimation errors to grow unbounded. Making a complex dynamical model of the AGV will not reduce this problem but in fact make it worse, as this model not only will contain the same errors, but most likely also introduce further inaccuracies caused by Coulomb friction, uncertain motor dynamics, etc.

Consider the case where the quantities k_r, k_l and b are imprecisely modelled by the quantities k_r^*, k_l^* and b^*:

$$k_r = \delta_r k_r^*, \; k_l = \delta_l k_l^* \text{ and } b = \delta_b b^* \quad (13)$$

Augment now the state vector with the three uncertain parameters:

$$x_{aug} = [X \; Y \; \theta \; \delta_r \; \delta_l \; \delta_b]^T \quad (14)$$

Instead of using equations (1) and (2) use:

$$\Delta d = \frac{\hat{\delta}_r k_r^* \alpha_r + \hat{\delta}_l k_l^* \alpha_l}{2} \quad (15)$$

$$\Delta\theta_k = \frac{\hat{\delta}_r k_r^* \alpha_r - \hat{\delta}_l k_l^* \alpha_l}{\hat{\delta}_b b^*} \quad (16)$$

The vision measurements will gradually improve the estimates of $\hat{\delta}_r$, $\hat{\delta}_l$, $\hat{\delta}_b$ and thereby improve the model used for the time updates. The new linearized system matrix, A_{aug} and the noise input matrix, G, are derived in Appendix A. A_{aug} and Q_{aug} takes the form:

$$A_{aug} = \begin{bmatrix} A & F \\ 0 & I \end{bmatrix}, \quad Q_{aug} = \begin{bmatrix} Q & 0 \\ 0 & S \end{bmatrix} \qquad (17)$$

The matrix F is a transition matrix linking the augmented states to the location estimate. The matrix S is a fictitious noise injection on the augmented states to ensure that these wont converge to a false estimate. This matrix should be chosen diagonal with eigenvalues small enough to reduce noise sensitivity and large enough to ensure convergence.

4 Simulations

Obviously noisy vision measurements or poor observability of the three augmented states can ruin the convergence of these and make the described approach useless. Even in ideal conditions with no noise on the vision measurements the observability of the augmented states depends on the trajectory the AGV follows. To verify the filter performance simulations using an advanced Simulink model of the AGV, contemplating both linear and non-linear friction forces and AGV dynamics has been performed. The Simulink model is described in [7].

The modelled AGV is a non-holomonic, three-wheeled robot, mounted with a camera able to detect artificial guide marks, see figure 1. Delays in

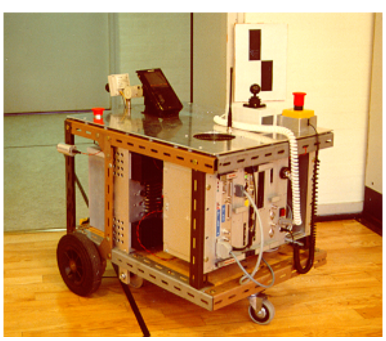

Figure 1: The AGV going through a door.

the vision measurements are accounted for using the method described in [8]. When calibrating the AGV only one guide mark is used. This enables us to place the reference system in the center of this guide mark and thereby know the location "exactly". Using additional guide marks could (if not placed accurately) introduce unnecessary errors.

Firstly, the filter is run for the AGV moving back and forth facing a guide mark as shown in figure 2. If a perfect pinhole camera with infinite resolution

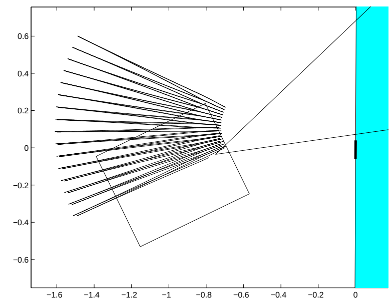

Figure 2: Trajectory of AGV facing a guide mark.

providing one (delayed) image per seconds, the convergence for the three augmented states are shown in figure 3. The true values in this experiment were

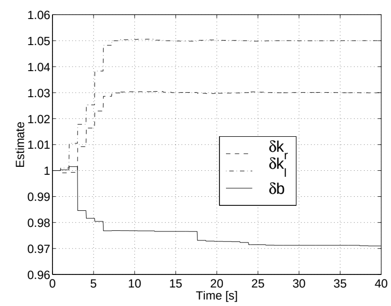

Figure 3: Estimate convergence with ideal camera.

$\delta_r = 1.03$, $\delta_l = 1.05$ and $\delta_b = 0.97$. The constants used in the filter therefore all deviates about 3-5% initially, while after 30 seconds the values are almost perfect. The mean values in the last 200 samples (i.e. 8 seconds) are:

$$E\{\hat{\delta}_r\} = 1.02997, \; E\{\hat{\delta}_l\} = 1.05000$$
$$E\{\hat{\delta}_b\} = 0.97097 \qquad (18)$$

So under noise free conditions augmenting the state improves the model by several orders of magnitude. This proves that the simple odometric model is quite accurate and that the parameter observability is good with the chosen trajectory.

The estimation error on the location is shown in figure 4. If the filter is not augmented the estimation error develops as shown in figure 5. From figure 4 and figure 5 it is seen that the non-augmented

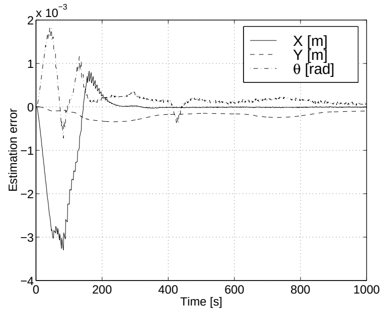

Figure 4: Estimation error for augmented filter.

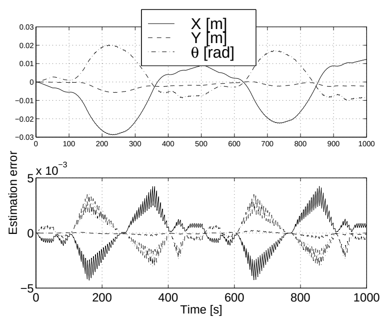

Figure 5: Estimation error for non-augmented filter. In the bottom plot fictitious process noise is injected.

filter performs a lot worse than the augmented. Fictitious noise injection helps somewhat but cannot outcompensate the effects of the modelling errors. Even here, the modelling error will drive the estimate further and further away from the actual state. The main reason that the error does not grow higher is that the AGV keeps changing direction and thereby outcompensates the systematic errors.

When a more realistic camera model with distortion and finite resolution is used the convergence of the parameter estimates are as shown in figure 6. Convergence here is much slower and the estimates never settles. The mean values of the estimates are quite good though:

$$E\{\hat{\delta}_r\} = 1.0315, \; E\{\hat{\delta}_l\} = 1.0479$$
$$E\{\hat{\delta}_b\} = 0.9726 \tag{19}$$

Because the initial errors on the estimates were so high we were forced to have a high gain from the vision residuals to the estimates. This was done by choosing the diagonal elements in $Q^{'}$ rather high. After this experiment one can improve the initial guesses by a factor of 10 and reduce the eigenvalues

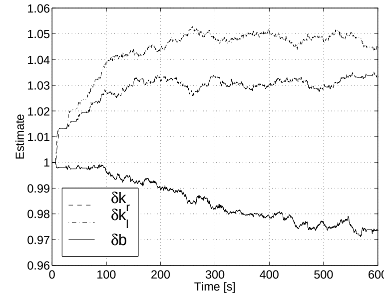

Figure 6: Estimate convergence with noisy camera.

of the $Q^{'}$ matrix accordingly. The results from this experiment are shown in figure 7.

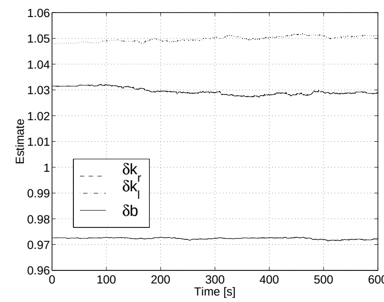

Figure 7: Estimate convergence with noisy camera.

The estimates remain rather constant but does improve somewhat. The mean values of the parameters in the last 200 samples are:

$$E\{\hat{\delta}_r\} = 1.0294, \; E\{\hat{\delta}_l\} = 1.0498$$
$$E\{\hat{\delta}_b\} = 0.9724 \tag{20}$$

The error on the estimate is seen to be less than half a percent now.

It is obvious that the location estimate behaves better with the augmented filter than a non-augmented with parameter errors as high as 3-5%. But does it perform better than a non-augmented filter using the relatively accurate parameters from (19)? If not, the augmented filter should only be used occasionally as an initial calibration of the computationally cheaper non-augmented filter. If however, somehow the parameters are changing during the operation of the AGV, for instance if the load on the AGV changes and the wheel radiuses therefore change, the augmented Kalman filter should be used always. Further, by using the augmented filter it is possible that some other slow varying changes on the AGV or the ground surface can be adapted into the parameters which might prove advantageous.

5 Experiments

The augmented filter is now tested on the real AGV. As the augmented filter is quite noise sensitive an initial experiment is made where a carefully calibrated surveillance camera is used as the vision source. First an initial identification experiment is made to check whether the parameters used in the filter actually matches the real AGV parameters, see fig 8. It is seen that the radius parameters con-

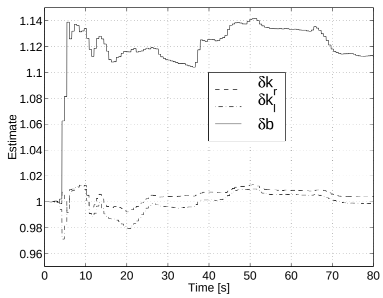

Figure 8: Estimate convergence in real life using surveillance camera.

verge to values within a half percent of the modelled values. The wheel base converges to a value almost 10 percent off. This is surprisingly high (almost 5 cm) and it is hard to believe that this is the actual physical parameter. As mentioned in the previous section it is possible that some other factors have been modelled into this parameter (calibration error on the surveillance camera etc).

Using these newly found parameters to recalibrate the odometry model an experiment is now performed where the filter uses wrong initial estimates for k_r, k_l and b of 3%, 5% and -3% respectively (as in section 4), see figure 9. It is seen that

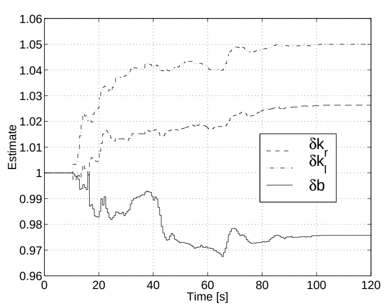

Figure 9: Estimate convergence in real life using surveillance camera.

the estimates converges after 100 seconds (or a total of 200 vision measurements) to the values:

$$E\{\hat{\delta}_r\} = 1.0263,\ E\{\hat{\delta}_l\} = 1.0500$$
$$E\{\hat{\delta}_b\} = 0.9757 \qquad (21)$$

The errors on the estimates are less than 0.5% here. Even if the wheel distance is not the physical one it is therefore still possible to detect changes in the parameters with great accuracy. The convergence time here is about 100 seconds, which can seem rather long. However, if the quality or frequency of the vision measurements is increased or the initial parameter error reduced, convergence can of course be obtained much faster. Additionally the trajectory the AGV follows affects the convergence time significantly. To be able to observe all the parameters a persistently exciting control signal must be applied.

Now an experiment using the rather noisy guide mark detection with the camera mounted on the AGV is performed. As the quality of especially the angular readings are a little dubious, the wheel distance parameter δ_b doesn't converge proberly. Therefore this parameter is fixed and only the two wheel radiuses are considered. The results are shown on figure 10. The estimates converge after

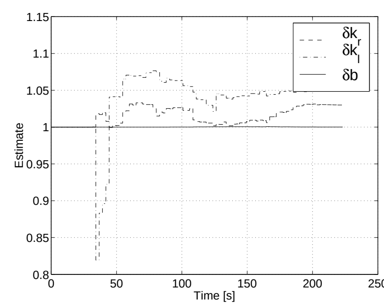

Figure 10: Estimate convergence in real life using guide marks. Only the radiuses are estimated.

150 seconds (or a total of 67 vision measurements) to the values:

$$E\{\hat{\delta}_r\} = 1.0300,\ E\{\hat{\delta}_l\} = 1.0500 \qquad (22)$$

So despite the poor observability of the wheel distance when the guide mark measurements are used, these can still provide very good estimates of the wheel radiuses.

6 Conclusion

In this paper an augmented Kalman filter for an AGV with a dual encoder system supported by vision measurements was designed. The filter model was a simple odometric model of the AGV augmented with three states for on-line estimation of uncertain parameters. This model was verified through simulations and experiments on a real AGV. These experiments show that it is possible to estimate the wheel radiuses and distance with a sub-percent accuracy if the quantity and quality of the vision measurements are sufficiently high.

In applications where the wheel radiuses change, for instance due to changes in load, the augmented filter can be very useful as a location estimator. Besides, the obtained accuracies also suggest that the filter is a worthy alternative for existing manual calibration methods for AGVs.

Appendix A

From (3)-(5) it is clear that:

$$A = \begin{bmatrix} 1 & 0 & a_{13} \\ 0 & 1 & a_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$a_{13} = \frac{\partial X}{\partial \Delta\theta} = -\Delta d \sin(\theta + \frac{\Delta\theta}{2})$$

$$a_{23} = \frac{\partial Y}{\partial \Delta\theta} = \Delta d \cos(\theta + \frac{\Delta\theta}{2})$$

The two inputs to the filter are given by:

$$\Delta d = \frac{\hat{\delta}_r k_r^* \alpha_r + \hat{\delta}_l k_l^* \alpha_l}{2}$$

$$\Delta\theta_k = \frac{\hat{\delta}_r k_r^* \alpha_r - \hat{\delta}_l k_l^* \alpha_l}{\hat{\delta}_b b^*}$$

Using these and defining $\phi = \theta + \frac{\Delta\theta}{2}$, the coefficients in the transition matrix F from equation (17) can be found:

$$f_{11} = \frac{\partial X}{\partial \delta_r} = \frac{k_r^* \alpha_r}{2} \cos\phi - \Delta d \frac{k_r^* \alpha_r}{\hat{\delta}_b b^*} \sin\phi$$

$$f_{12} = \frac{\partial X}{\partial \delta_l} = \frac{k_l^* \alpha_l}{2} \cos\phi + \Delta d \frac{k_l^* \alpha_l}{\hat{\delta}_b b^*} \sin\phi$$

$$f_{13} = \frac{\partial X}{\partial \delta_b} = \Delta d \frac{\Delta\theta}{2\hat{\delta}_b} \sin\phi$$

$$f_{21} = \frac{\partial Y}{\partial \delta_r} = \frac{k_r^* \alpha_r}{2} \sin\phi + \Delta d \frac{k_r^* \alpha_r}{\hat{\delta}_b b^*} \cos\phi$$

$$f_{22} = \frac{\partial Y}{\partial \delta_l} = \frac{k_l^* \alpha_l}{2} \sin\phi - \Delta d \frac{k_l^* \alpha_l}{\hat{\delta}_b b^*} \cos\phi$$

$$f_{23} = \frac{\partial Y}{\partial \delta_b} = -\Delta d \frac{\Delta\theta}{2\hat{\delta}_b} \cos\phi$$

$$f_{31} = \frac{\partial \theta}{\partial \delta_r} = \frac{k_r^* \alpha_r}{\hat{\delta}_b b^*}$$

$$f_{32} = \frac{\partial \theta}{\partial \delta_l} = -\frac{k_l^* \alpha_l}{\hat{\delta}_b b^*}$$

$$f_{33} = \frac{\partial \theta}{\partial \delta_b} = -\frac{\Delta\theta}{\hat{\delta}_b}$$

The noise input matrix, G is:

$$G = \begin{bmatrix} \cos\phi & -\frac{1}{2}\Delta d \sin\phi \\ \sin\phi & \frac{1}{2}\Delta d \cos\phi \\ 0 & 1 \end{bmatrix}$$

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